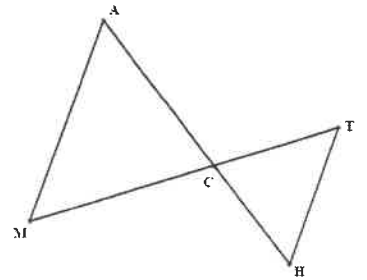


2017 John O'Bryan Mathematical Competition
Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. When solved for x , the equation $3(4x - 7) + 6 = 12(x + k)$ has infinitely many solutions. Find k . Give your answer as a fraction reduced to simplest form.
2. The points $(7, a)$ and $(b, 6)$ lie on a line with the equation $x - 3y + 25 = 0$. Find $\sqrt{3a + b}$.
3. Let A be a positive two-digit integer. The integer B is the same as A when its digits are reversed. What is the largest value of A such that $A = 3B - 2$?
4. The ratio of two supplementary angles is 1:8. Find the degree measure of the smaller of the two angles.

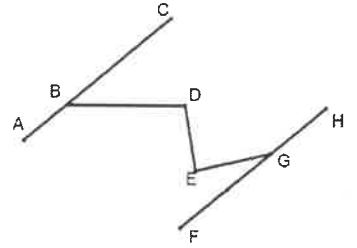
5. In the diagram at the right, $\overline{MA} \parallel \overline{TH}$, $MA = 4x$, $AC = 5x + 2$, $MC = 3x + 4$, $CT = x + 3$ and $TH = x + 2$. Find the length of \overline{CH} .



6. \overline{AB} is a chord in circle O such that the degree measure of minor arc \widehat{AB} is one-quarter the degree measure of major arc \widehat{AB} . Find the degree measure of $\angle OAB$.
7. The average of three values is $3x + 2y$. If two of the values are $4x + 2y$ and $3x - y$, find the third value. Give your answer as an expression in terms of x and/or y .
8. When eight coins are flipped, find the probability that tails occurs exactly three times. Write your answer as a common fraction reduced to lowest terms.
9. A right triangle has acute angles such that the degree measure of one is half of the degree measure of the other. The area of this triangle is $32\sqrt{3}$. Find the length of the hypotenuse of this triangle.
10. A rectangle has diagonal $2\sqrt{13}$ and one side that is 2 more than another. Find the length of the shortest side of this rectangle.
11. How many of the first 10000 natural numbers are divisible by all of the values 2, 3, 4, 5, 6, 7, 8, and 9?
12. The numeric value of the expression $2017 + k$ is a multiple of 9, where k is a positive integer less than 50. How many possible values are there for k ?

13. The sum of the smallest and largest of three consecutive even integers is 52. The integer k is 10 less than the median of the three consecutive even integers. If $p^2 = k$ where p is a positive integer, find the value of p .

14. In the diagram to the right, $\overline{AC} \parallel \overline{HF}$, $\angle CBD \cong \angle GED$, the measure of $\angle E$ is 18° less than $\angle D$, and the measure of $\angle FGE$ is 44° . Find the degree measure of $\angle D$.

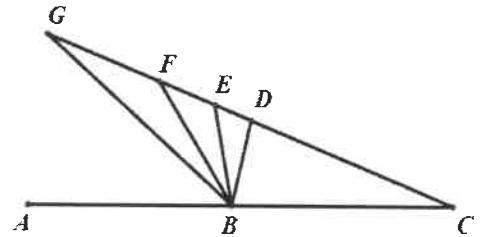


15. A circle with area 81π is inscribed in an equilateral triangle. Find the area of the triangle, rounding your answer to the nearest integer.

16. If $3x^2 + 8 = 56$, find the smallest possible value of $4x - 5$.

17. If $\begin{bmatrix} 3 & 2 \\ d & 4 \end{bmatrix} + 2\begin{bmatrix} 4 & 3 \\ e & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ 4 & c \end{bmatrix}$, find the value of the expression $(a + b + c + 2d + 4e)$.

18. In the diagram at the right, the measure of $\angle CBF$ is 73° more than the measure of $\angle ABG$. \overline{BD} bisects $\angle CBG$ and \overline{BE} and \overline{BF} trisect $\angle DBG$. Find the degree measure of $\angle EBC$.



19. One of the following statements is selected at random. Find the probability that the selected statement is true for all parallelograms. Write your answer as a common fraction reduced to lowest terms.

- The opposite angles are supplementary.
- The opposite angles are congruent.
- Any pair of consecutive angles are supplementary.
- The opposite sides are congruent.
- The diagonals are congruent.
- The diagonals are perpendicular to each other.
- The diagonals bisect the vertex angles.
- The diagonals bisect each other.

20. Max runs into his friends Zoe and Alex on a particular day of the week. He knows that Zoe lies on Mondays, Tuesdays and Wednesdays, and tells the truth on the other days of the week. Alex, on the other hand, lies on Thursdays, Fridays, and Saturdays, but tells the truth on other days of the week. Zoe says "Yesterday was one of my lying days." Alex responds "Yesterday was one of my lying days". What day of the week is this particular day? Write the whole word for this day of the week.

Name: _____ **ANSWERS** _____

Team Code: _____

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1. $-\frac{5}{4}$ Must be this reduced fraction.

11. 3

2. 5

12. 5

3. 82

13. 4

4. 20

14. 80

5. 6

15. 421

6. 54

16. -21

7. $2x + 5y$ OR $5y + 2x$

17. 41

8. $\frac{7}{32}$ Must be this reduced fraction.

18. 92 Degrees Optional

9. 16

19. $\frac{1}{2}$

10. 4

20. Thursday Must be this entire word.

Awards Lists and Solutions to the Team Competition may be found at
<http://math.nku.edu/job>